① $\begin{vmatrix} 1 & 2 & 1 \\ 1 & -1 & 1 \\ a & 1 & -1 \end{vmatrix} = 1+2a+1-(-a+1-2) = 3a+3$ $\underline{a \neq -1}$

② $\lim\limits_{x \to 0} \left(\frac{x+2}{x+1}\right)^{\frac{\sin 3x}{x}} = 2^3 = \underline{8}$

$\lim\limits_{x \to \infty} \frac{\ln(x+1)}{2+e^{-x}} = \left(\frac{\infty}{2+0}\right) = \underline{\infty}$

③ $D(f) = \mathbb{R}$

$y' = \frac{2}{3} \cdot \frac{1}{\sqrt[3]{x}} + \frac{2}{3} = \frac{2}{3} \frac{1+\sqrt[3]{x}}{\sqrt[3]{x}}$

x	$(-\infty,-1)$	-1	$(-1,0)$	0	$(0,\infty)$
f'	$+$	0	$-$	$\nexists$	$+$
f	↗	$\frac{1}{3}$ LOK. MAX.	↘	0 LOK. MIN.	↗

$y' = 0 \iff x = -1$

y' nedefinirano $\iff x = 0$

④ $D(f) = \mathbb{R}$ ABS nema'

AS v smeru ∞: $k = \lim\limits_{x \to \infty} \operatorname{arccotg} x = 0$

$\underline{q = 1}$ $\quad q = \lim\limits_{x \to \infty} x \cdot \operatorname{arccotg} x = (\infty \cdot 0) = \lim\limits_{x \to \infty} \frac{\operatorname{arccotg} x}{\frac{1}{x}} \overset{L'H}{=} \ldots = 1$

AS v smeru $-\infty$: $k = \lim\limits_{x \to -\infty} \operatorname{arccotg} x = \pi$

$\underline{y = \pi x + 1}$ $\quad q = \lim\limits_{x \to -\infty} x \cdot \operatorname{arccotg} x - \pi x = \lim\limits_{x \to -\infty} \frac{\operatorname{arccotg} x - \pi}{\frac{1}{x}} \overset{L'H}{=} \ldots = 1$

⑤ $f: y = x^2 - x + 3$

$p: y = 4 - x \ldots k_p = -1$

$k_t = -\frac{1}{k_p} = 1$

$t \perp p$

$y' = 2x - 1$

$y'(x_0) = k$

$2x_0 - 1 = 1$

$x_0 = 1$

$t: y - y_0 = k(x - x_0)$

$y - 3 = 1 \cdot (x - 1)$

$\underline{0 = x - y + 2}$

⑥ $\frac{\partial f}{\partial x} = 3x^2 - 6y$ $\quad 3(x^2 - 2y) = 0 \Rightarrow y = \frac{x^2}{2}$

$\frac{\partial f}{\partial y} = -6x + 24y^2$ $\quad 6(4y^2 - x) = 0$

$4 \cdot \left(\frac{x^2}{2}\right)^2 - x = 0$

$x(x^3 - 1) = 0 \quad \begin{cases} x_1 = 0 & y_1 = 0 \\ x_2 = 1 & y_2 = \frac{1}{2} \end{cases}$

$A_1[0,0]$

$A_2[1, \frac{1}{2}]$

$\frac{\partial^2 f}{\partial x^2} = 6x \qquad \frac{\partial^2 f}{\partial y^2} = 48y \qquad \frac{\partial^2 f}{\partial x \partial y} = \frac{\partial^2 f}{\partial y \partial x} = -6$

$D_2 = \begin{vmatrix} 6x & -6 \\ -6 & 48y \end{vmatrix} = 6x \cdot 48y - (-6)^2 = 36(8xy - 1)$

$D_2(A_1) < 0$ .. A_1 nie je extrém

$D_2(A_2) > 0$ .. A_2 je extrém . $\frac{\partial^2 f}{\partial x^2}(A_2) > 0$ .. A_2 je lokálne minimum

⑦ $TC(x) = 2\,800x + 24\,000$

$TR(x) = p \cdot x$ $\qquad TR(20) > TC(20)$

$20p > 2800 \cdot 20 + 24\,000$

$20p > 80\,000$

$p > 4\,000$

Podnikateľ musí daný výrobok predávať za cenu väčšiu ako 4 000.