① $\int e^{\sqrt{x}}\,dx = \left| \begin{matrix} x = t^2 \\ dx = 2t\,dt \end{matrix} \right| = \int e^t \cdot 2t\,dt = \left| \begin{matrix} u = 2t & u' = 2 \\ v' = e^t & v = e^t \end{matrix} \right| = 2t\cdot e^t - \int 2\cdot e^t\,dt$

$= 2t\cdot e^t - 2\cdot e^t + C = 2\cdot e^t(t-1) + C = \underline{2\cdot e^{\sqrt{x}}\cdot(\sqrt{x}-1) + C}$

② $\int \frac{1}{x^3+x^2+x}\,dx = \int \frac{1}{x\cdot(x^2+x+1)}\,dx = \int \frac{1}{x}\,dx - \int \frac{x+1}{x^2+x+1}\,dx =$

$= \ln|x| - \frac{1}{2}\int \frac{2x+2}{x^2+x+1}\,dx =$

$= \ln|x| - \frac{1}{2}\int\left(\frac{2x+1}{x^2+x+1} + \frac{1}{(x+\frac{1}{2})^2+\frac{3}{4}}\right)dx =$

$= \ln|x| - \frac{1}{2}\ln|x^2+x+1| - \frac{1}{2}\cdot\frac{1}{\sqrt{\frac{3}{4}}}\,\mathrm{arctg}\,\frac{x+\frac{1}{2}}{\sqrt{\frac{3}{4}}} + C$

$= \underline{\ln\frac{x}{\sqrt{x^2+x+1}} - \frac{1}{\sqrt{3}}\,\mathrm{arctg}\,\frac{2x+1}{\sqrt{3}} + C}$

$\frac{1}{x(x^2+x+1)} = \frac{A}{x} + \frac{Bx+C}{x^2+x+1}$

$1 = A(x^2+x+1) + (Bx+C)\cdot x$

$1 = Ax^2 + Ax + A + Bx^2 + Cx$

$x^2:\ 0 = A + B$

$x^1:\ 0 = A + C$

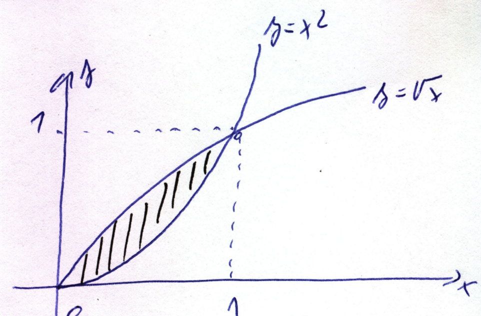
$x^0:\ 1 = A \quad \Rightarrow A = 1 \quad C = -1 \quad B = -1$

③ $\left.\begin{matrix} y = \sqrt{x} \\ y = x^2 \end{matrix}\right\}$ $\sqrt{x} = x^2 \quad x \geq 0$

$x = x^4$

$0 = x^4 - x$

$0 = x(x^3-1) \quad x_{1,2} = \begin{cases} 0 & y_1 = 0 \\ 1 & y_2 = 1 \end{cases}$



$V = \pi\cdot\int_0^1 (\sqrt{x})^2 - (x^2)^2\,dx = \pi\int_0^1 x - x^4\,dx = \pi\left[\frac{x^2}{2} - \frac{x^5}{5}\right]_0^1 = \pi\left(\frac{1}{2} - \frac{1}{5}\right) = \underline{\frac{3}{10}\pi}$

④ $\sum_{n=1}^{\infty}\left(\frac{n+3}{n+1}\right)^{n^2}$ Cauchyho odmocninové kritérium

$\lim_{n\to\infty}\sqrt[n]{\left(\frac{n+3}{n+1}\right)^{n^2}} = \lim_{n\to\infty}\left(\frac{n+3}{n+1}\right)^n =_{"} 1^{\infty"} = \lim_{n\to\infty}\left(1 + \frac{2}{n+1}\right)^n =$

$$= \lim_{n\to\infty} \left(1+\frac{1}{\frac{n+1}{2}}\right)^{\frac{n+1}{2}\cdot n\cdot\frac{2}{n+1}} = \lim_{n\to\infty}\left[\left(1+\frac{1}{\frac{n+1}{2}}\right)^{\frac{n+1}{2}}\right]^{\frac{2n}{n+1}} = e^2 > 1$$

diverguje

• môžeme použiť aj nutnú podmienku konvergencie $-\lim_{n\to\infty}\left(\frac{n+3}{n+1}\right)^{n^2} = \infty \neq 0$

diverguje

5. $e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \dots$

$$e^{-x^2} = 1 + (-x^2) + \frac{(-x^2)^2}{2!} + \frac{(-x^2)^3}{3!} + \frac{(-x^2)^4}{4!} + \dots = 1 - x^2 + \frac{x^4}{2} - \frac{x^6}{6} + \dots$$

$$\int_0^{0,1} e^{-x^2}\,dx = \int_0^{0,1}\left(1 - x^2 + \frac{x^4}{2} - \frac{x^6}{6} + \dots\right)dx = \left[x - \frac{x^3}{3} + \frac{x^5}{10} - \frac{x^7}{42} + \dots\right]_0^{0,1} =$$

$$= 0{,}1 - \frac{0{,}1^3}{3} + \frac{0{,}1^5}{10} - \frac{0{,}1^7}{42} + \dots =$$

$$= 0{,}1 - 0{,}000333333 + 0{,}000001 - 0{,}000000002 + \dots =$$

$$= 0{,}099667665$$

(6) $(1+y^2) + x \cdot y \cdot \frac{dy}{dx} = 0 \quad / \cdot \frac{1}{x} \cdot \frac{1}{1+y^2}$

$$\frac{1}{x}\,dx + \frac{y}{1+y^2}\,dy = 0 \quad / \cdot 2$$

$$\int \frac{2}{x}\,dx + \int \frac{2y}{1+y^2}\,dy = \ln c$$

$$2 \cdot \ln x + \ln|1+y^2| = \ln c$$

$$\ln\left(x^2 \cdot (1+y^2)\right) = \ln c$$

$$x^2 \cdot (1+y^2) = c$$

(7) $y'' + 6y' + 9y = x \cdot e^x$

$$y = y_h + y_p$$

$$\lambda^2 + 6\lambda + 9 = 0$$

$$(\lambda+3)^2 = 0$$

$$\lambda_{1,2} = -3$$

$$y_h = C_1 \cdot e^{-3x} + C_2 \cdot x \cdot e^{-3x}$$

$$y_p = (ax+b) \cdot e^x$$

$$y_p' = a \cdot e^x + (ax+b) \cdot e^x = (ax+a+b) \cdot e^x$$

$$y_p'' = a \cdot e^x + (ax+a+b) \cdot e^x = (ax+2a+b) \cdot e^x$$

$$(ax+2a+b) \cdot e^x + 6 \cdot (ax+a+b) \cdot e^x + 9 \cdot (ax+b) \cdot e^x = x e^x$$

$$ax + 2a + b + 6ax + 6a + 6b + 9ax + 9b = x$$

$$x^1: \ a + 6a + 9a = 1$$

$$x^0: \ 2a + b + 6a + 6b + 9b = 0$$

$$16a = 1 \quad \Rightarrow a = \frac{1}{16}$$

$$8a + 16b = 0 \qquad \frac{1}{2} + 16b = 0 \Rightarrow b = -\frac{1}{32}$$

$$y_p = \left(\frac{1}{16}x - \frac{1}{32}\right) \cdot e^x$$

$$y = C_1 \cdot e^{-3x} + C_2 \cdot x \cdot e^{-3x} + \left(\frac{1}{16}x - \frac{1}{32}\right) \cdot e^x$$