① $\begin{pmatrix} 1 & 2 \\ 2 & 4 \end{pmatrix} \cdot X = \begin{pmatrix} 5 \\ 10 \end{pmatrix}$ $\quad A \cdot X = B \quad |A| = 0$, A^{-1} neexistuje

$2\times 2 \cdot 2\times 1 = 2\times 1$

$\begin{pmatrix} 1 & 2 \\ 2 & 4 \end{pmatrix} \cdot \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 5 \\ 10 \end{pmatrix}$ $\quad \begin{pmatrix} x_1 + 2x_2 \\ 2x_1 + 4x_2 \end{pmatrix} = \begin{pmatrix} 5 \\ 10 \end{pmatrix}$

$x_1 + 2x_2 = 5$

$2x_1 + 4x_2 = 10$

$x_1 = 5 - 2x_2$

nekonečne veľa riešení

$X = \begin{pmatrix} 5-2x_2 \\ x_2 \end{pmatrix}$

② a) $\lim\limits_{x\to\infty} \left(\frac{3x+1}{x+2}\right)^{\frac{\ln x}{x}} = 3^0 = 1$

$\lim\limits_{x\to\infty} \frac{\ln x}{x} = \text{„}\frac{\infty}{\infty}\text{“} = \lim\limits_{x\to\infty} \frac{\frac{1}{x}}{1} = 0$

b) $\lim\limits_{x\to\infty} \left(\frac{1+5x}{x+2}\right)^{\ln(1+e^{-x})} = 5^0 = 1$

③ $f(x) = x^{\sin x} - \frac{\pi}{2} + \frac{x \cdot \sin x}{\operatorname{arctg}(x^2)} \cdot \log 2$

• $f_1(x) = x^{\sin x} = e^{\sin x \cdot \ln x}$ $\quad f_1(x)' = e^{\sin x \cdot \ln x} \cdot \left(\cos x \cdot \ln x + \sin x \cdot \frac{1}{x}\right) =$

$= x^{\sin x} \cdot \left(\cos x \cdot \ln x + \sin x \cdot \frac{1}{x}\right)$

• $f_2(x) = \frac{\pi}{2}$ $\quad f_2(x)' = 0$

• $f_3(x) = \frac{x \cdot \sin x}{\operatorname{arctg}(x^2)} \cdot \log 2$ $\quad f_3'(x) = \frac{(\sin x + x\cos x) \cdot \operatorname{arctg}(x^2) - x \cdot \sin x \cdot \frac{1 \cdot 2x}{1+x^4}}{\operatorname{arctg}^2(x^2)} \cdot$

④ $MR(x) = TR'(x)$

$TR(x) = p \cdot x = \left(450 - \frac{x}{10}\right) \cdot x$

$TR(x) = 450x - \frac{x^2}{1}$

$MR(x) = 450 - \frac{x}{5}$

(4.) $y = \dfrac{2x^2+3x}{x+1} + e^{-x}$ $\qquad D(f) = (-\infty, -1) \cup (-1, \infty)$

• ABS $x = -1$ je asymptota

$$\lim_{x \to -1^+} \left(\frac{2x^2+3x}{x+1} + e^{-x} \right) = -\infty$$

$$\lim_{x \to -1^-} \left(\quad -||- \quad \right) = \infty$$

• ASeS v smere ∞: $\qquad k = \lim_{x \to \infty} \left(\dfrac{2x^2+3x}{x^2+x} + \dfrac{e^{-x}}{x} \right) = 2$

$y = 2x+1$

$$q = \lim_{x \to \infty} \left(\frac{2x^2+3x}{x+1} + e^{-x} - 2x \right) = \lim_{x \to \infty} \left(\frac{x}{x+1} + e^{-x} \right) = 1$$

v smere $-\infty$: $\qquad k = 2$

neexistuje $\qquad q = \infty$

(5) $f'(x) = -6\sin x - 6\cos x - 3x^2 + 6x + 6$ $\qquad f'(0) = 0$

$f''(x) = -6\cos x + 6\sin x - 6x + 6$ $\qquad f''(0) = 0$

$f'''(x) = 6\sin x + 6\cos x - 6$ $\qquad f'''(0) = 0$

$f^{IV}(x) = 6\cos x - 6\sin x$ $\qquad f^{IV}(0) = 6 \neq 0$ $\quad$ 4-párne je tu extrém

(6) $f(x,y) = x^3 - 2y^2 + xy - 11x + 6y + 2$

$\dfrac{\partial f}{\partial x} = 3x^2 + y - 11$ $\qquad x = 6 + 4y$ $\qquad y = 11 - 3x^2$ $\qquad x_{1,2} = \dfrac{-1 \pm 49}{24} = \left\langle \begin{matrix} 2 \\ -\frac{25}{12} \end{matrix} \right.$

$\dfrac{\partial f}{\partial y} = -4y + x - 6$ $\qquad$ 3. $\qquad -4(11 - 3x^2) + x - 6 = 0$

$12x^2 + x - 50 = 0$ $\qquad y_1 = -1$

$y_2 = -\dfrac{97}{48}$

$\dfrac{\partial^2 f}{\partial x^2} = 6x$ $\qquad D_2 = \begin{vmatrix} 6x & 1 \\ 1 & -4 \end{vmatrix} = -24x - 1$ $\qquad A_1[2; -1]$ $\qquad A_2\left[-\frac{25}{12}; -\frac{97}{48}\right]$

$\dfrac{\partial^2 f}{\partial y^2} = -4$

$D_2(A_1) < 0$ $\quad$ A_1 nie je extrém

$\dfrac{\partial^2 f}{\partial x \partial y} = 1$ $\qquad D_2(A_2) > 0$ $\quad$ A_2 je extrém $\quad \dfrac{\partial^2 f}{\partial x^2}(A_2) < 0$ $\quad$ lok. maximum