1.
$$\begin{pmatrix} 1 & 1 & 1 & 4 \\ 1 & 2 & -1 & 3 \\ 1 & 0 & 3 & 5 \end{pmatrix} \sim \begin{pmatrix} 1 & 1 & 1 & 4 \\ 0 & 1 & -2 & -1 \\ 0 & -1 & 2 & 1 \end{pmatrix} \sim \begin{pmatrix} 1 & 0 & 3 & 5 \\ 0 & 1 & -2 & -1 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

$x_3 = t$

$x_2 = -1 + 2t$

$x_1 = 5 - 3t$

$(5-3t;\ -1+2t;\ t),\ t \in R$

2.

$$\lim_{x \to 0} (1+x)^{\frac{1}{\sin 5x}} = (1^{\infty}) = \lim_{x \to 0} e^{\frac{1}{\sin 5x} \cdot \ln(1+x)} = e^{\frac{1}{5}}$$

$$\lim_{x \to 0} \frac{\ln(1+x)}{\sin 5x} = \left(\frac{0}{0}\right) = \lim_{x \to 0} \frac{\frac{1}{1+x}}{\cos 5x \cdot 5} = \frac{1}{5}$$

$$\lim_{x \to \infty} \frac{\operatorname{arctg} x + \ln x}{e^{1-x} - 2} = \left(\frac{\frac{\pi}{2} + \infty}{0 - 2}\right) = -\infty$$

3.

$f(x) = x^5 - 5x^4 + 5x^3 + 1 \qquad x \in \langle -2; 3 \rangle$

$f'(x) = 5x^4 - 20x^3 + 15x^2$

$5x^2(x^2 - 4x + 3) = 0$

$x_1 = 0$

$x_2 = 1$

$x_3 = 3$

$f(0) = 1$

$f(1) = 2$

$f(3) = -26$

$f(-2) = -151$

v -2 je minimum -151

v 1 je maximum 2

4.

$f_1(x) = \ln \sqrt{x^2 + \sin 4x}$

$f_2(x) = x^{\cos x} = e^{\cos x \cdot \ln x}$

$f_3(x) = \dfrac{x \cdot e^x}{\operatorname{arctg}(3x)}$

$$f_1'(x) = \frac{1}{\sqrt{x^2 + \sin 4x}} \cdot \frac{1}{2 \cdot \sqrt{x^2 + \sin 4x}} \cdot (2x + \cos 4x \cdot 4)$$

$$f_2'(x) = x^{\cos x} \cdot \left(-\sin x \cdot \ln x + \cos x \cdot \frac{1}{x}\right)$$

$$f_3'(x) = \frac{(e^x + x \cdot e^x) \cdot \operatorname{arctg}(3x) - x \cdot e^x \cdot \frac{1}{1 + (3x)^2} \cdot 3}{\operatorname{arctg}^2(3x)}$$

5. $f(x) = e^{\frac{1}{x-2}}$; $D(f) = (-\infty, 2) \cup (2, \infty)$

$$f'(x) = e^{\frac{1}{x-2}} \cdot \left(\frac{-1}{(x-2)^2} \right)$$

$$f''(x) = e^{\frac{1}{x-2}} \cdot \left(\frac{-1}{(x-2)^2} \right)^2 + e^{\frac{1}{x-2}} \cdot \frac{2}{(x-2)^3} = e^{\frac{1}{x-2}} \cdot \frac{1+2(x-2)}{(x-2)^4} =$$

$$= e^{\frac{1}{x-2}} \cdot \frac{2x-3}{(x-2)^4}$$

$f''(x) = 0 \iff x = \frac{3}{2}$

x	$(-\infty, \frac{3}{2})$	$\frac{3}{2}$	$(\frac{3}{2}; 2)$	$(2, \infty)$
$f''(x)$	$-$	0	$+$	$+$
$f(x)$	$\cap$	I.B.	$\cup$	$\cup$

6. $f(x,y) = 2x^3 - xy^2 + 5x^2 + y^2$

$$\frac{\partial f}{\partial x} = 6x^2 - y^2 + 10x$$

$$\frac{\partial f}{\partial y} = -x \cdot 2y + 2y$$

$6x^2 - y^2 + 10x = 0$

$2y(1-x) = 0$

$y = 0$

$6x^2 + 10x = 0$

$2x(3x+5) = 0$

$x = 0$

$x = -\frac{5}{3}$

$x = 1$

$6 - y^2 + 10 = 0$

$y^2 = 16$

$y = \pm 4$

S.B: $A_1[0, 0]$

$A_2[-\frac{5}{3}; 0]$

$A_3[1; 4]$

$A_4[1; -4]$

$$\frac{\partial^2 f}{\partial x^2} = 12x + 10$$

$$\frac{\partial^2 f}{\partial y^2} = -2x + 2$$

$$\frac{\partial^2 f}{\partial x \partial y} = -2y$$

$$D_2 = \begin{vmatrix} 12x+10 & -2y \\ -2y & 2-2x \end{vmatrix} = 4 \cdot (6x+5)(1-x) - (-2y)^2$$

$D_2(A_1) = 4 \cdot 5 \cdot 1 - 0 > 0$ je extrém, lok. minimum

$D_2(A_2) = 4 \cdot (-5)(\frac{8}{3}) < 0$ nie je extrém

$D_2(A_3) = D_2(A_4) = 0 - 16 < 0$ nie je extrém

(7) $TPr(x) = (-2x+90) \cdot x - (3x^2+15) = -5x^2+90x-15$

$TPr'(x) = -10x+90$

$-10x+90=0$

$\underline{\underline{x=9}}$