(1.) $|A| = 1 + 2a + 1 - (-a + 1 - 2) = 3a + 3 \qquad \underline{\underline{a \neq -1}}$

(2.) • $\lim\limits_{x \to 0} (\cos 4x)^{\frac{1}{x^2}} \quad „1^{\infty}" = \lim\limits_{x \to 0} e^{\frac{1}{x^2} \cdot \ln(\cos 4x)} = \underline{\underline{e^{-8}}}$

$$\lim_{x \to 0} \frac{\ln(\cos 4x)}{x^2} = „\frac{0}{0}" = \lim_{x \to 0} \frac{\frac{1}{\cos 4x} \cdot (-\sin 4x) \cdot 4}{2x} =$$

$$= -2 \cdot \lim_{x \to 0} \frac{1}{\cos 4x} \cdot \lim_{x \to 0} \frac{\sin 4x}{x} = -2 \cdot 1 \cdot 4 = -8$$

• $\lim\limits_{x \to \infty} \frac{e^x + x}{e^{-x} + 1} = „\frac{\infty + \infty}{0 + 1}" = \underline{\underline{\infty}}$

(3) $D(f) = (0,1) \cup (1, \infty)$

$$f'(x) = \frac{\ln x - 1}{\ln^2 x}$$

$$f''(x) = \frac{\frac{1}{x} \cdot \ln^2 x - (\ln x - 1) \cdot 2 \cdot \ln x \cdot \frac{1}{x}}{\ln^4 x} =$$

$$= \frac{\ln x - 2(\ln x - 1)}{x \cdot \ln^3 x} = \frac{2 - \ln x}{x \cdot \ln^3 x}$$

$f''(x) = 0 \iff x = e^2$

$f''(x)$ neexistuje $\times$

x	$(0,1)$	$(1,e^2)$	e^2	(e^2,∞)
$f''(x)$	$-$	$+$	0	$-$
$f(x)$	$\cap$	$\cup$	I.B.	$\cap$

(4.) $D(f) = (-\infty, 0) \cup (0, \infty)$

• ABS $x = 0$ $\quad \lim\limits_{x \to 0^+} x \cdot \operatorname{arctg} \frac{2}{x} = 0 \cdot \frac{\pi}{2} = 0$

není ABS $\quad \lim\limits_{x \to 0^-} x \cdot \operatorname{arctg} \frac{2}{x} = 0 \cdot \left(-\frac{\pi}{2}\right) = 0$

ASoS v nevl. $+\infty$ $\quad k = \lim\limits_{x \to \infty} \operatorname{arctg} \frac{2}{x} = 0$

$y = 2$

$$q = \lim_{x \to \infty} x \cdot \operatorname{arctg} \frac{2}{x} = „\infty \cdot 0" = \lim_{x \to \infty} \frac{\operatorname{arctg} \frac{2}{x}}{\frac{1}{x}} = „\frac{0}{0}" =$$

$$\lim_{x \to \infty} \frac{\frac{1}{1 + (\frac{2}{x})^2} \cdot \frac{-2}{x^2}}{-\frac{1}{x^2}} = 2$$

v smere $-\infty$: $k = \lim\limits_{x \to -\infty} \operatorname{arctg} \frac{2}{x} = 0$

$q = 2$

$$q = \lim_{x \to -\infty} x \cdot \operatorname{arctg} \frac{2}{x} = \lim_{x \to -\infty} \frac{\operatorname{arctg} \frac{2}{x}}{\frac{1}{x}} = \lim_{x \to -\infty} \frac{\frac{1}{1+\left(\frac{2}{x}\right)^2} \cdot \frac{-2}{x^2}}{-\frac{1}{x^2}} = 2$$

(5) $D(f) = \mathbb{R}$

$$f'(x) = \frac{2}{3} x^{-\frac{1}{3}} - \frac{2}{3} = \frac{2}{3}\left(\frac{1}{\sqrt[3]{x}} - 1\right) = \frac{2}{3} \frac{1 - \sqrt[3]{x}}{\sqrt[3]{x}}$$

$f'(x) = 0 \iff x = 1$

$f'(x)$ neexistuje $\Rightarrow x = 0$

x	$(-\infty, 0)$	0	$(0, 1)$	1	$(1, \infty)$
$f'(x)$	$-$	N	$+$	0	$-$
$f(x)$	$\searrow$	lok.	$\nearrow$	lok.	$\searrow$

minimum $f(0) = 0$

maximum $f(1) = \frac{1}{3}$

(6) $\frac{\partial f}{\partial x} = \frac{4}{x} - y^2$

$x > 0$

$\frac{\partial f}{\partial y} = -2xy + 2y$

$\frac{\partial^2 f}{\partial x^2} = -\frac{4}{x^2}$

$\frac{\partial^2 f}{\partial y^2} = -2x + 2$

$\frac{\partial^2 f}{\partial x \partial y} = -2y$

$\frac{4}{x} - y^2 = 0$

$2y(1 - x) = 0$

$y = 0$ nevyhovuje

$x = 1 \quad y = \pm 2$

$A_1[1; 2]$

$A_2[1; -2]$

$$D_2 = \begin{vmatrix} -\frac{4}{x^2} & -2y \\ -2y & 2 - 2x \end{vmatrix} = -\frac{4 \cdot 2}{x^2}(1 - x) - (4y^2)$$

$D_2(A_1) = D_2(A_2) = -16$ funkcia nemá lokálne extrémy

(7) $TPr \geq 10\,000$

$TPr = 45x - (20x + 250)$

$25x - 250 \geq 10\,000$

$\underline{\underline{x \geq 410}}$