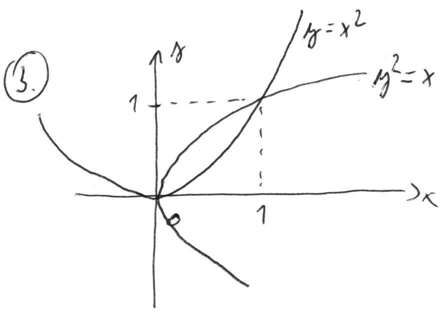
1. $$\int x\cdot e^{2x}\,dx = \left| \begin{array}{ll} u = x & u' = 1 \\ v' = e^{2x} & v = \frac{1}{2}e^{2x} \end{array} \right| = \frac{x}{2}\cdot e^{2x} - \frac{1}{2}\int e^{2x}\,dx = \frac{x}{2}\cdot e^{2x} - \frac{1}{2}\cdot\frac{1}{2}e^{2x} + c$$

$$= \frac{1}{4}e^{2x}(2x-1) + c$$

2. $$\int \frac{1}{x^3+x}\,dx = \int \frac{1}{x\cdot(x^2+1)}\,dx = \int\left(\frac{1}{x} - \frac{x}{x^2+1}\right)dx = \ln|x| - \frac{1}{2}\ln|x^2+1| + c =$$

$$= \ln\left|\frac{x}{\sqrt{x^2+1}}\right| + c$$

3. 

$$\left.\begin{array}{l} y = x^2 \\ y^2 = x \end{array}\right\} (x^2)^2 = x$$

$$x^4 - x = 0$$

$$x(x^3-1) = 0 \iff x_{1,2} = \left\langle \begin{array}{l} 0 \\ 1 \end{array}\right.$$

$$S = \int_0^1 (\sqrt{x} - x^2)\,dx = \left[\frac{x^{\frac{3}{2}}}{\frac{3}{2}} - \frac{x^3}{3}\right]_0^1 = \left[\frac{2}{3}\cdot\sqrt{x^3} - \frac{1}{3}x^3\right]_0^1 = \frac{2}{3} - \frac{1}{3} = \frac{1}{3}$$

4. $$\sum_{n=0}^{\infty} \frac{(2n)!}{(n!)^2}$$

d'Alembertovo kritérium: $$\lim_{n\to\infty} \frac{\frac{[2(n+1)]!}{[(n+1)!]^2}}{\frac{(2n)!}{(n!)^2}} = \lim_{n\to\infty} \frac{(n!)^2\cdot(2n+2)(2n+1)(2n)!}{[(n+1)\cdot n!]^2\cdot(2n)!} =$$

$$= \lim_{n\to\infty} \frac{(2n+2)\cdot(2n+1)}{(n+1)^2} = 4 > 1$$ rad DIVERGUJE

(5) $\sum_{n=2}^{\infty} \frac{1}{n^2-n} = \sum_{n=2}^{\infty} \frac{1}{n\cdot(n-1)} = \sum_{n=2}^{\infty} \left(\frac{1}{n-1} - \frac{1}{n}\right)$

$$s_n = 1 - \cancel{\frac{1}{2}} +$$
$$+ \cancel{\frac{1}{2}} - \cancel{\frac{1}{3}} +$$
$$+ \cancel{\frac{1}{3}} - \cancel{\frac{1}{4}} +$$
$$+ \cancel{\frac{1}{4}} - \cancel{\frac{1}{5}} +$$
$$\vdots$$
$$+ \frac{1}{n-1} - \frac{1}{n} = 1 - \frac{1}{n}$$

$$s = \lim_{n\to\infty} s_n = \lim_{n\to\infty} \left(1 - \frac{1}{n}\right) = 1 \qquad \sum_{n=2}^{\infty} \frac{1}{n^2-n} = 1$$

(6) $x \cdot y \cdot y' = y^2 + 3$

$$x \cdot y\, dy - (y^2+3)\,dx = 0$$

$$\frac{2y\,dy}{y^2+3} - \frac{2}{x}\,dx = 0$$

$$\int \frac{2y\,dy}{y^2+3} - \int \frac{2}{x}\,dx = \ln c$$

$$\ln|y^2+3| - 2\cdot \ln|x| = \ln c$$

$$\ln \frac{y^2+3}{x^2} = \ln c$$

$$y^2 + 3 = c x^2 \qquad y^2 = cx^2 - 3$$

(7) $y'' + 2y' + y = x$

$$\lambda^2 + 2\lambda + 1 = 0$$
$$(\lambda+1)^2 = 0$$
$$\lambda_{1,2} = -1$$

$$y_h = C_1 \cdot e^{-x} + C_2\, x \cdot e^{-x}$$

$$y_p = ax + b$$
$$y_p' = a$$
$$y_p'' = 0$$

$$0 + 2\cdot(a) + (ax+b) = x$$
$$ax + 2a + b = x$$

$$\left.\begin{aligned} a &= 1 \\ 2a + b &= 0 \end{aligned}\right\} \; b = -2$$

$$y_p = x - 2$$

$$y = y_h + y_p$$

$$y = C_1 e^{-x} + C_2 x e^{-x} + x - 2$$