(1) $\int \frac{x}{\cos^2 x}dx = \left| \begin{matrix} u = x & u' = 1 \\ v' = \frac{1}{\cos^2 x} & v = \operatorname{tg} x \end{matrix} \right| = x\operatorname{tg}x - \int \frac{\sin x}{\cos x}dx = \underline{\underline{x \cdot \operatorname{tg} x + \ln|\cos x| + c}}$

(2) $\int_0^\infty \frac{3}{x^3+1}dx = \lim_{c\to\infty} \int_0^c \frac{3}{x^3+1}dx = *$

$$\frac{3}{(x+1)(x^2-x+1)} = \frac{A}{x+1} + \frac{Bx+C}{x^2-x+1}$$

$$\left.\begin{matrix} x^2: \ 0 = A+B \\ x^1: \ 0 = -A+B+C \\ x^0: \ 3 = A+C \end{matrix}\right\} \quad \begin{matrix} A=1 \\ B=-1 \\ C=2 \end{matrix}$$

$$3 = A(x^2-x+1) + (Bx+C)\cdot(x+1)$$

$$3 = Ax^2 - Ax + A + Bx^2 + Bx + Cx + C$$

$$* = \lim_{c\to\infty} \int_0^c \left(\frac{1}{x+1} + \frac{-x+2}{x^2-x+1}\right)dx = \lim_{c\to\infty}\left(\int_0^c \frac{1}{x+1}dx - \frac{1}{2}\int_0^c \frac{2x-4}{x^2-x+1}dx\right) =$$

$$= \lim_{c\to\infty}\left(\int_0^c \frac{1}{x+1}dx - \frac{1}{2}\int_0^c \frac{2x-1-3}{x^2-x+1}dx\right) = \lim_{c\to\infty}\Bigg[\ln(x+1) - \frac{1}{2}\ln|x^2-x+1| +$$

$$+ \frac{3}{2}\frac{1}{\sqrt{\frac{3}{4}}}\operatorname{arctg}\frac{x-\frac{1}{2}}{\frac{\sqrt{3}}{2}}\Bigg]_0^c = \lim_{c\to\infty}\left[\ln\left|\frac{x+1}{\sqrt{x^2-x+1}}\right| + \sqrt{3}\operatorname{arctg}\frac{2x-1}{\sqrt{3}}\right]_0^c =$$

$$= \lim_{c\to\infty}\left\{\left(\ln\frac{c+1}{\sqrt{c^2-c+1}} + \sqrt{3}\operatorname{arctg}\frac{2c-1}{\sqrt{3}}\right) - \left(\ln\frac{1}{1} + \sqrt{3}\operatorname{arctg}\frac{-1}{\sqrt{3}}\right)\right\} =$$

$$= \sqrt{3}\cdot\frac{\pi}{2} - \sqrt{3}\left(-\frac{\pi}{6}\right) = \sqrt{3}\left(\frac{\pi}{2}+\frac{\pi}{6}\right) = \underline{\underline{\frac{2\cdot\sqrt{3}}{3}\pi}}$$

(3) $y = \frac{2}{3}\cdot\sqrt{x^3} = \frac{2}{3}\cdot x^{\frac{3}{2}}$

$y' = \frac{2}{3}\cdot\frac{3}{2}\cdot x^{\frac{1}{2}} = \sqrt{x}$

$$l = \int_0^1 \sqrt{1+(\sqrt{x})^2}\,dx = \int_0^1 \sqrt{1+x}\,dx = \left| \begin{matrix} 1+x = t^2 \\ dx = 2t\,dt \end{matrix} \right.$$

$$= \int_1^{\sqrt{2}} t\cdot 2t\,dt = \left[2\frac{t^3}{3}\right]_1^{\sqrt{2}} = \frac{2}{3}\left(\sqrt{2}^3 - 1\right) =$$

$$= \underline{\underline{\frac{2}{3}\left(2\cdot\sqrt{2}-1\right)}}$$

④ $\sum_{n=1}^{\infty} \frac{\sqrt{n}}{1+3n^2}$

LIMITNÉ POROVNÁVACIE KRITÉRIUM: $\lim_{n\to\infty} \frac{\frac{\sqrt{n}}{1+3n^2}}{\frac{\sqrt{n}}{n^2}} = \lim_{n\to\infty} \frac{n^2}{1+3n^2} = \frac{1}{3}$

$\sum_{n=1}^{\infty} \frac{\sqrt{n}}{n^2} = \sum_{n=1}^{\infty} \frac{1}{n^{3/2}}$ KONVERGUJE

$\Downarrow$ $\sum_{n=1}^{\infty} \frac{\sqrt{n}}{1+3n^2}$ KONVERGUJE

⑤ $s = \sum_{n=1}^{\infty} \frac{(x+1)^n}{n}$

$s' = \sum_{n=1}^{\infty} (x+1)^{n-1}$

$s' = \frac{1}{1-(x+1)} = -\frac{1}{x}$ pre $x \in (-2;0)$

$s = \int -\frac{1}{x}dx = -\ln|x| + c$

$s(-1) = 0$

$-\ln|-1| + c = 0$

$c = 0$

$s = -\ln|x|$

$x=0$ $\sum_{n=1}^{\infty} \frac{1}{n}$ DIVERGUJE

$x=-2$ $\sum_{n=1}^{\infty} \frac{(-1)^n}{n}$ KONVERGUJE

$K = \langle -2; 0)$

⑥ $y' - \frac{2}{x+1} \cdot y = (x+1)^3$, $y(0) = 1$ LINEÁRNA

$y' - \frac{2}{x+1} y = 0$

$\frac{1}{y} dy - \frac{2}{x+1} dx = 0$

$\int \frac{1}{y} dy - 2\int \frac{1}{x+1} dx = \ln c$

$\ln|y| - 2\ln|x+1| = \ln c$

$\ln \frac{y}{(x+1)^2} = \ln c$

$y = c \cdot (x+1)^2$

$y = c(x)\cdot(x+1)^2 \qquad y' = c'(x)\cdot(x+1)^2 + c(x)\cdot 2\cdot(x+1)$

$$c'(x)\cdot(x+1)^2 + \cancel{2c(x)(x+1)} - \cancel{\frac{2}{x+1}\cdot c(x)\cdot(x+1)^2} = (x+1)^3$$

$$c'(x)\cdot(x+1)^2 = (x+1)^3$$

$$c'(x) = x+1$$

$$c(x) = \int(x+1)\,dx = \frac{1}{2}(x+1)^2 + C$$

$$y = \left(\frac{1}{2}(x+1)^2 + C\right)\cdot(x+1)^2$$

$$y = C\cdot(x+1)^2 + \frac{1}{2}(x+1)^4 \qquad y(0) = 1$$

$$1 = C + \frac{1}{2} \Rightarrow C = \frac{1}{2}$$

$$\underline{\underline{y = \frac{1}{2}(x+1)^2 + \frac{1}{2}(x+1)^4}}$$

(2) $y'' + 2y' + y = e^{-x}$

$\lambda^2 + 2\lambda + 1 = 0$

$(\lambda+1)^2 = 0$

$\lambda_{1,2} = -1$

$y_h = C_1\cdot e^{-x} + C_2\cdot x\cdot e^{-x}$

$y_p = a\cdot e^{-x}\cdot x^2$

$y_p' = a\cdot(-e^{-x}\cdot x^2 + e^{-x}\cdot 2x) = a e^{-x}(2x - x^2)$

$y_p'' = a\cdot(-e^{-x}\cdot(2x - x^2) + e^{-x}(2-2x)) =$

$= a\cdot e^{-x}(-2x + x^2 + 2 - 2x) = a\cdot e^{-x}\cdot(2 - 4x + x^2)$

$$a\cdot e^{-x}\cdot(2 - 4x + x^2) + 2\cdot a e^{-x}(2x - x^2) + a e^{-x}\cdot x^2 = e^{-x}$$

$$a\cdot e^{-x}(2 - 4x + x^2 + 4x - 2x^2 + x^2) = e^{-x}$$

$$2a = 1$$

$$a = \frac{1}{2} \qquad y_p = \frac{1}{2}x^2\cdot e^{-x}$$

$$y = (C_1 + C_2 x)e^{-x} + \frac{x^2}{2}\cdot e^{-x}$$