① $$\int \frac{\cos x}{\sin^2 x - 5\sin x + 6}\,dx = \left|\begin{matrix}\sin x = t\\ \cos x\,dx = dt\end{matrix}\right| = \int \frac{dt}{t^2-5t+6} = \int \frac{dt}{(t-2)(t-3)} =$$

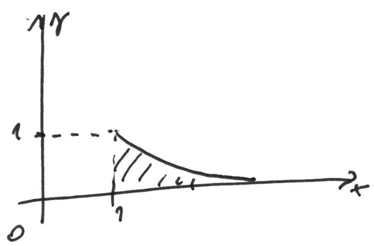
$$= \int \frac{1}{t-3}\,dt + \int \frac{1}{t-2}\,dt = \ln|t-3| - \ln|t-2| + c = \underline{\ln\left|\frac{\sin x - 3}{\sin x - 2}\right| + c}$$

② $$\int \frac{x}{\sqrt{x^2-x}}\,dx = \int \frac{x-\frac{1}{2}+\frac{1}{2}}{\sqrt{x^2-x}}\,dx = \frac{1}{2}\int \frac{2x-1}{\sqrt{x^2-x}}\,dx + \frac{1}{2}\int \frac{1}{\sqrt{x^2-x}}\,dx = *$$

$$\frac{1}{2}\int \frac{2x-1}{\sqrt{x^2-x}}\,dx = \left|\begin{matrix}x^2-x = t^2\\ (2x-1)\,dx = 2t\,dt\end{matrix}\right| = \frac{1}{2}\int \frac{2t\,dt}{t} = \int dt = t = \sqrt{x^2-x}$$

$$* = \underline{\sqrt{x^2-x} + \frac{1}{2}\ln\left|x-\frac{1}{2}+\sqrt{x^2-x}\right| + c}$$

③



$$V = \pi\int_1^\infty \left(\frac{1}{x}\right)^2 dx = \pi \lim_{c\to\infty}\int_1^c x^{-2}\,dx = \pi \lim_{c\to\infty}\left[-\frac{1}{x}\right]_1^c =$$

$$= \pi\cdot\lim_{c\to\infty}\left(-\frac{1}{c}+1\right) = \underline{\pi}$$

④ $$\sum_{n=1}^{\infty} 2^{-\ln n}$$

d'Alambert: $$\lim_{n\to\infty}\frac{2^{-\ln(n+1)}}{2^{-\ln(n)}} = \lim_{n\to\infty} 2^{\ln n - \ln(n+1)} =$$

$$= \lim_{n\to\infty} 2^{\ln\frac{n}{n+1}} = 1$$

Raabe: $$\lim_{n\to\infty} n\left(1-2^{\ln\frac{n}{n+1}}\right) = \lim_{n\to\infty}\frac{1-2^{\ln\frac{n}{n+1}}}{\frac{1}{n}} =$$

$$\overset{LH}{=} \lim_{n\to\infty}\frac{-2^{\ln\frac{n}{n+1}}\cdot\ln 2\cdot\frac{n+1}{n}\cdot\frac{n+1-n}{(n+1)^2}}{-\frac{1}{n^2}} = \ln 2\cdot\lim_{n\to\infty} 2^{\ln\frac{n}{n+1}}\cdot\frac{n^2}{n\cdot(n+1)} =$$

$$= \underline{\ln 2 < 1}$$

DIVERGENTE

5. $$\int_0^{0,1} \frac{\sin x}{x}\,dx = \int_0^1 \frac{\sum_{k=0}^{\infty} (-1)^k \cdot \frac{x^{2k+1}}{(2k+1)!}}{x}\,dx = \int_0^1 \sum_{k=0}^{\infty} (-1)^k \cdot \frac{x^{2k}}{(2k+1)!}\,dx =$$

$$= \left[\sum_{k=0}^{\infty} (-1)^k \cdot \frac{x^{2k+1}}{(2k+1)\cdot(2k+1)!}\right]_0^{0,1} = \sum_{k=0}^{\infty} (-1)^k \cdot \frac{(0,1)^{2k+1}}{(2k+1)(2k+1)!} - 0 =$$

$$= 0,1 - \frac{0,1^3}{3.3!} + \frac{0,1^5}{5.5!} - \frac{0,1^7}{7.7!} + \dots = 0,1 - 0,0000555556 +$$

$$+ 0,0000000167 - \dots = \underline{0,0999444611}$$

6. $$(x^2+y^2)\cdot y' = 2xy \quad /\cdot \frac{1}{x^2}$$

homogénna: $z = \frac{y}{x}$

$$\left(1+\frac{y^2}{x^2}\right)\cdot y' - 2\cdot\frac{y}{x} = 0$$

$y = z\cdot x$

$y' = z'\cdot x + z$

$$(1+z^2)\cdot(z'x+z) - 2z = 0$$

$$(1+z^2)\cdot z'x + z + z^3 - 2z = 0$$

$$(1+z^2)\cdot dz\,x + (z^3-z)\,dx = 0$$

$$\frac{z^2+1}{z^3-z}\,dz + \frac{1}{x}\,dx = 0$$

$$\int \frac{z^2+1}{z^3-z}\,dz + \int \frac{1}{x}\,dx = \ln c$$

$$\int \frac{1}{z-1}\,dz + \int \frac{1}{z+1}\,dz - \int \frac{1}{z}\,dz + \int \frac{1}{x}\,dx = \ln c$$

$$\ln \frac{(z-1)(z+1)}{z}\cdot x = \ln c$$

$$\frac{z^3+1}{z(z-1)(z+1)} = \frac{A}{z} + \frac{B}{z-1} + \frac{C}{z+1}$$

$$z^2+1 = A(z-1)(z+1) + Bz(z+1) + Cz(z-1)$$

$z=0$: $1=-A$ $\quad A=-1$

$z=1$: $2=2B$ $\quad B=1$

$z=-1$: $2=2C$ $\quad C=1$

$$\frac{k^2-1}{k}\cdot x = c$$

$$\frac{\frac{y^2}{x^2}-1}{\frac{y}{x}}\cdot x = c$$

$$\frac{\frac{y^2-x^2}{x^2}}{\frac{y}{x}}\cdot x = C$$

$$\frac{y^2-x^2}{y} = C \qquad \boxed{y^2-x^2=Cy}$$

(7) $y''-2y' = x+e^{3x}$

$\lambda^2-2\lambda=0$

$\lambda(\lambda-2)=0$

$\lambda_{1,2} = \begin{cases} 0 \\ 2 \end{cases}$

$y_h = C_1+C_2\cdot e^{2x}$

$y''-2y'=x$

$y_{p1}=(ax+b)\cdot x$

$y_{p1}=ax^2+bx$

$y_{p1}'=2ax+b$

$y_{p1}''=2a$

$2a-2\cdot(2ax+b)=x$

$2a-4ax-2b=x$

$-4a=1$

$2a-2b=0$

$a=-\frac{1}{4}$

$b=-\frac{1}{4}$

$y_{p1}=-\frac{1}{4}x^2-\frac{1}{4}x$

$y''-2y'=e^{3x}$

$y_{p2}=a\cdot e^{3x}$

$y_{p2}'=3ae^{3x}$

$y_{p2}''=9ae^{3x}$

$9ae^{3x}-2\cdot 3ae^{3x}=e^{3x}$

$3a=1$

$a=\frac{1}{3}$

$y_{p2}=\frac{1}{3}e^{3x}$

$$\boxed{y=C_1+C_2e^{2x}-\frac{1}{4}x^2-\frac{1}{4}x+\frac{1}{3}e^{3x}}$$