(1) $$\int \frac{1}{\cos^2 x \cdot \sin^2 x}\,dx = \int \frac{\cos^2 x + \sin^2 x}{\cos^2 x \cdot \sin^2 x}\,dx = \int \frac{1}{\sin^2 x}\,dx + \int \frac{1}{\cos^2 x}\,dx =$$

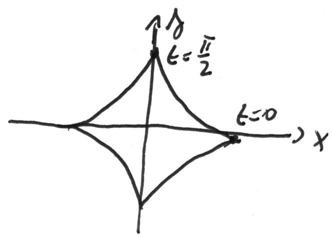
$$= -\operatorname{cotg} x + \operatorname{tg} x + C$$

(2) $$\int_0^4 e^{\sqrt{x}}\,dx = \left| \begin{matrix} x = t^2 \\ dx = 2t\,dt \end{matrix} \right| = \int_0^2 e^t \cdot 2t\,dt = \left| \begin{matrix} u = 2t & u' = 2 \\ v' = e^t & v = e^t \end{matrix} \right| =$$

$$= \left[2t \cdot e^t\right]_0^2 - \int_0^2 2e^t\,dt = 4 \cdot e^2 - 2\left[e^t\right]_0^2 = 4e^2 - 2(e^2 - 1) = 2e^2 + 2$$

(3) $x = \cos^3 t$

$y = \sin^3 t$



$$V = \pi \cdot \int_{t_1}^{t_2} y^2 \cdot |x'|\,dt$$

$$V = 2\pi \int_0^{\frac{\pi}{2}} (\sin^3 t)^2 \cdot \left|3 \cdot \cos^2 t \cdot (-\sin t)\right| dt =$$

$$= 6\pi \int_0^{\frac{\pi}{2}} \sin^7 t \cdot \cos^2 t\,dt = 6\pi \int_0^{\frac{\pi}{2}} (1 - \cos^2 t)^3 \cdot \cos^2 t \cdot \sin t\,dt = \left| \begin{matrix} \cos t = r \\ -\sin t\,dt = dr \end{matrix} \right| =$$

$$= 6\pi \int_1^0 (1 - r^2)^3 \cdot r^2 (-dr) = 6\pi \int_0^1 (1 - 3r^2 + 3r^4 - r^6) \cdot r^2\,dr =$$

$$= 6\pi \int_0^1 (r^2 - 3r^4 + 3r^6 - r^8)\,dr = 6\pi \left[\frac{r^3}{3} - 3\frac{r^5}{5} + 3\frac{r^7}{7} - \frac{r^9}{9}\right]_0^1 =$$

$$= 6\pi \left(\frac{1}{3} - \frac{3}{5} + \frac{3}{7} - \frac{1}{9}\right) = 6\pi \cdot \frac{16}{315}$$

(4) $\sum_{n=1}^{\infty} \frac{1}{4n^2-1}$ $\qquad \frac{1}{4n^2-1} = \frac{1}{(2n-1)(2n+1)} = \frac{\frac{1}{2}}{2n-1} - \frac{\frac{1}{2}}{2n+1}$

$$s_n = \frac{1}{2} \cdot \left\{ \frac{1}{1} - \cancel{\frac{1}{3}} + \cancel{\frac{1}{3}} - \cancel{\frac{1}{5}} + \cancel{\frac{1}{5}} - \cancel{\frac{1}{7}} + \dots + \cancel{\frac{1}{2n-1}} - \frac{1}{2n+1} \right\} = \frac{1}{2} \left\{ 1 - \frac{1}{2n+1} \right\}$$

$$s = \lim_{n\to\infty} s_n = \lim_{n\to\infty} \frac{1}{2}\left(1 - \frac{1}{2n+1}\right)$$

$$s = \frac{1}{2}$$

(5) $\sum_{n=0}^{\infty} \frac{(2x-1)^n}{n+2} = \sum_{n=0}^{\infty} \frac{[2\cdot(x-\frac{1}{2})]^n}{n+2} = \sum_{n=0}^{\infty} \frac{2^n}{n+2} \cdot (x-\frac{1}{2})^n$

$a_0 = \frac{1}{2}$

$a_n = \frac{2^n}{n+2}$

$$L = \lim_{n\to\infty} \frac{a_{n+1}}{a_n} = \lim_{n\to\infty} \frac{\frac{2^{n+1}}{n+1+2}}{\frac{2^n}{n+2}} = \lim_{n\to\infty} \frac{\cancel{2^n}\cdot 2\cdot(n+2)}{\cancel{2^n}\cdot(n+3)} =$$

$$= \lim_{n\to\infty} \frac{2n+4}{n+3} = 2 \qquad r = \frac{1}{L} = \frac{1}{2}$$

$x \in (a_0 - r, a_0 + r)$

$x \in (0; 1)$

$x = 1$ $\quad \sum_{n=0}^{\infty} \frac{1}{n+2}$ diverguje (porovnávacie kritérium)

$x = 0$ $\quad \sum_{n=0}^{\infty} \frac{(-1)^n}{n+2}$ konverguje (Leibnizovo kritérium) $\qquad K = \langle 0; 1)$

⑥ $(x^2+y^2)\cdot y' - 2xy = 0 \;/\cdot \frac{1}{x^2}$

$\left(1+\left(\frac{y}{x}\right)^2\right)\cdot y' - 2\cdot\frac{y}{x} = 0$ $\qquad r = \frac{y}{x} \qquad y = r\cdot x \qquad y' = r'\cdot x + r$

$(1+r^2)\cdot(r'x+r) - 2r = 0$

$(1+r^2)\cdot r'x + r + r^3 - 2r = 0$

$(1+r^2)\cdot r'x + r^3 - r = 0$

$\frac{r^2+1}{r^3-r}\,dr + \frac{1}{x}\,dx = 0$

$\int\left(\frac{1}{r-1} + \frac{1}{r+1} - \frac{1}{r}\right)dr + \int\frac{1}{x}\,dx = \ln c$

$\ln\frac{(r-1)\cdot(r+1)}{r} + \ln x = \ln c$

$\frac{r^2-1}{r}\cdot x = c$

$\frac{\frac{y^2}{x^2}-1}{\frac{y}{x}}\cdot x = c$

$\frac{\frac{y^2-x^2}{x^2}}{\frac{y}{x}}\cdot x = c$

$y^2 - x^2 = cy$

⑦ $y'' + y = 6\cdot e^x + \sin x$

$\lambda^2 + 1 = 0$

$\lambda_{1,2} = \pm i$

$y_h = C_1\cdot\cos x + C_2\cdot\sin x$

$y'' + y = 6\cdot e^x$

$y_{p1} = a\cdot e^x$

$y_{p1}' = y_{p1}'' = a e^x$

$a\cdot e^x + a e^x = 6 e^x$

$2a = 6$

$a = 3$

$y_{p1} = 3e^x$

$y'' + y = \sin x$

$y_{p2} = (a\sin x + b\cos x)\cdot x$

$y_{p2}' = (a\cos x - b\sin x)\cdot x +$

$+ (a\sin x + b\cos x) =$

$= (a - bx)\cdot\sin x + (b + ax)\cos x$

$y_{p2}'' = -b\cdot\sin x + (a - bx)\cos x +$

$+ a\cos x - (b + ax)\sin x =$

$= (-2b - ax)\sin x + (2a - bx)\cos x$

$$(-2b-ax)\sin x+(2a-bx)\cos x+ax\sin x+bx\cos x=\sin x$$

$$-2b\sin x+2a\cos x=\sin x$$

$\sin x: \quad -2b=1 \quad b=-\frac{1}{2}$

$\cos x: \quad 2a=0 \quad a=0$

$$y_{p2}=-\frac{1}{2}x\cos x$$

$$y=c_1\cdot\cos x+c_2\sin x+3e^x-\frac{1}{2}x\cos x$$