① $$\int \frac{\operatorname{arctg} x}{x^2}\,dx = \left| \begin{matrix} u = \operatorname{arctg} x & u' = \frac{1}{1+x^2} \\ v' = \frac{1}{x^2} & v = -\frac{1}{x} \end{matrix} \right| = -\frac{1}{x}\cdot\operatorname{arctg} x + \int \frac{1}{x\cdot(1+x^2)}\,dx =$$

$$= -\frac{1}{x}\cdot\operatorname{arctg} x + \int \frac{1+x^2-x^2}{x\cdot(1+x^2)}\,dx = -\frac{1}{x}\cdot\operatorname{arctg} x + \int \frac{1}{x}\,dx - \int \frac{x}{1+x^2}\,dx =$$

$$= -\frac{1}{x}\operatorname{arctg} x + \ln|x| - \frac{1}{2}\ln|1+x^2| + C$$

② $$\int \frac{1}{\cos^4 x\cdot \sin^2 x}\,dx = \left| \operatorname{tg} x = t \right| = \int \frac{1}{\frac{1}{(1+t^2)^2}\cdot\frac{t^2}{1+t^2}}\cdot\frac{dt}{1+t^2} = \int \frac{(1+t^2)^2}{t^2}\,dt =$$

$$= \int \frac{1+2t^2+t^4}{t^2}\,dt = \int\left(\frac{1}{t^2}+2+t^2\right)dt = -\frac{1}{t} + 2t + \frac{t^3}{3} =$$

$$= -\frac{1}{\operatorname{tg} x} + 2\operatorname{tg} x + \frac{\operatorname{tg}^3 x}{3} + C$$

③ $$l = \int_a^b \sqrt{1+(f'(x))^2}\,dx$$

$$l = \int_0^2 \sqrt{1+\left(\frac{3}{2}x^{\frac{1}{2}}\right)^2}\,dx = \int_0^2 \sqrt{1+\frac{9}{4}x}\,dx = \frac{1}{2}\int_0^2 \sqrt{4+9x}\,dx = \left| \begin{matrix} 4+9x = t^2 \\ 9dx = 2t\,dt \end{matrix} \right| =$$

$$= \frac{1}{2}\int_2^{\sqrt{22}} t\cdot\frac{2}{9}t\,dt = \frac{1}{9}\left[\frac{t^3}{3}\right]_2^{\sqrt{22}} = \frac{1}{27}\left(\sqrt{22}^3 - 8\right)$$

④ $$\lim_{n\to\infty}\left(\frac{3n-1}{4n+1}\right)^2 = \frac{9}{16} \neq 0$$ DIVERGUJE

⑤ $$e^x = \sum_{n=0}^{\infty}\frac{x^n}{n!} = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \dots$$

$$\sqrt[3]{e} = e^{\frac{1}{3}} = 1 + \frac{1}{3} + \frac{\left(\frac{1}{3}\right)^2}{2} + \frac{\left(\frac{1}{3}\right)^3}{6} + \dots \doteq 1 + \frac{1}{3} + \frac{1}{18} + \frac{1}{162} = 1{,}39506$$

⑥ $y' - y = x+1 \quad , \quad y(0) = 6$

lineárna: bez P.S.: $y' - y = 0$

$$\frac{1}{y} dy - dx = 0$$

$$\ln y - x = \ln c$$

$$\ln y + \ln e^{-x} = \ln c$$

$$y = c \cdot e^{x}$$

variácia konštánt

$$y = c(x) \cdot e^{x}$$

$$y' = c'(x) \cdot e^{x} + c(x) \cdot e^{x}$$

$$c'(x) \cdot e^{x} + c(x) \cdot e^{x} - c(x) \cdot e^{x} = x+1$$

$$c'(x) \cdot e^{x} = x+1$$

$$c(x) = \int (x+1) \cdot e^{-x} dx =$$

$$= \left| \begin{matrix} u = x+1 & u' = 1 \\ v' = e^{-x} & v = -e^{-x} \end{matrix} \right| = -(x+1) \cdot e^{-x} + \int e^{-x} dx$$

$$= -(x+1)e^{-x} - e^{-x} + C = -(x+2) \cdot e^{-x} + c$$

$$y = \left[-(x+2) \cdot e^{-x} + c \right] \cdot e^{x} =$$

$$y = c \cdot e^{x} - x - 2 \qquad y(0) = 6 \qquad 6 = c \cdot e^{0} - 0 - 2$$

$$c = 8$$

$$y = 8e^{x} - x - 2$$

⑦ $y'' - 2y' + y = e^{-x} + e^{x}$

$$\lambda^2 - 2\lambda + 1 = 0$$

$$(\lambda - 1)^2 = 0$$

$$\lambda_{1,2} = 1$$

$$y_h = C_1 \cdot e^{x} + C_2 x \cdot e^{x}$$

$$y'' - 2y' + y = e^{-x}$$

$$y_{p1} = a \cdot e^{-x}$$

$$y_{p1}' = -a e^{-x}$$

$$y_{p1}'' = a e^{-x}$$

$$a \cdot e^{-x} + 2a e^{-x} + a e^{-x} = e^{-x}$$

$$4a = 1$$

$$a = \frac{1}{4}$$

$$y_{p1} = \frac{1}{4} e^{-x}$$

$$y'' - 2y' + y = e^{x}$$

$$y_{p2} = a \cdot e^{x} \cdot x^2$$

$$y_{p2}' = a \cdot (e^{x} \cdot x^2 + e^{x} \cdot 2x) =$$

$$= a \cdot e^{x} (x^2 + 2x)$$

$$y_{p2}'' = a \cdot (e^{x} \cdot (x^2 + 2x) + e^{x}(2x+2))$$

$$= a \cdot e^{x} (x^2 + 4x + 2)$$

$$a \cdot e^{x} \left[x^2 + 4x + 2 - 2(x^2 + 2x) + x^2 \right] = e^{x}$$

$$2a = 1 \quad a = \frac{1}{2}$$

$$y_{p2} = \frac{1}{2} x^2 \cdot e^{x}$$

$$y = C_1 \cdot e^{x} + C_2 x \cdot e^{x} + \frac{1}{4} e^{-x} + \frac{1}{2} x^2 \cdot e^{x}$$